*chenia magna*, Owen, and dwelling on the evidences of a progress from a more generalized to a more specialized type of Ruminant dentition in the extinct Cameloid forms succeeding each other, from the old Pliocene of Nebraska to the new or Postpliocene of Mexico.

Tables of dimensions of teeth and vertebræ of *Palauchenia*, *Auchenia*, and *Camelus*, and drawings arranged for one folding and three 4to plates, accompany the memoir.

## III. "On the Proof of the Law of Errors of Observations." By M. W. Crofton, F.R.S. Received March 24, 1869.

(Abstract.)

The object of this Paper is to give the mathematical proof, in its most general form, of the law of single errors of observations, on the hypothesis that each error in practice arises from the joint operation of a large number of independent sources of error, each of which, did it exist alone, would occasion errors of extremely small amount as compared generally with those actually produced by all the sources combined. This proof is contained in a process given for a different object, namely, Poisson's generalization of Laplace's investigation of the law of the mean results of a large number of observations, to be found in the 'Connaissance des Temps' for 1827, and also in his 'Recherches sur la Probabilité des Jugements;' it is also reproduced in Mr. Todhunter's able 'History of the Theory of Probability.' It is not therefore pretended that any new results are arrived at in the present Paper. Considering, however, the importance and celebrity of the question, and the refined and difficult character of Poisson's analysis, it will not probably be deemed superfluous to show how the same law may be demonstrated with equal generality, in a much more simple and elementary manner. The difficulty of the general proof seems indeed to have been so extensively felt, that several attempts have been made to simplify it. However, so far as the present writer is aware, no proof has been given, except Poisson's, which is not open to grave objection, as based upon unjustifiable assumptions, or as unduly limiting the generality of the investigation.

The mathematical reasoning in this Paper is based entirely on the above-mentioned hypothesis as to the causation of error, namely, that errors *in rerum naturâ* result from the superposition of a large number of minuter errors arising from a number of independent sources. The laws of these elementary errors are supposed entirely unknown, no further restriction whatever being imposed on the generality of the investigation; as would be the case, for instance, were we to assume (as has sometimes been done) that each independent source gives positive and negative errors with equal facility. To decide fully how far the above hypothesis (which seems now to be generally accepted) really agrees with facts, is an extremely subtle question in

philosophy,—one which probably never can be more than partially resolved. Still, even a cursory and superficial examination of a few particular cases seems to show that, far from being a mere arbitrary assumption, it is at least a reasonable and probable account of what really does take place in nature, in many large classes of errors of observations. The history of practical astronomy, in particular, seems to prove that, whatever doubt may be entertained of its exactness as applied to the errors of rude and primitive observers, we may safely accept it in the case of the refined and delicate observations of modern astronomers.

It would be scarcely possible in this Abstract to convey any clear idea of the mathematical analysis employed in reducing the above hypothesis to calculation. It will suffice to remark that, whereas in the processes given by Laplace and Poisson, when applied to the problem before us, the elementary component errors are at first supposed of finite magnitude, and finite in number, and the results are afterwards modified for the supposition that the magnitude of the errors becomes infinitesimal and their number infinite; much simplicity is gained in this Paper by making these suppositions at the commencement. Also, instead of taking a simultaneous view of all the elementary errors, as affecting the actual or resultant error, the latter is considered as produced by the superposition of some one of the elementary errors upon the error produced by the combination of all the others. We are thus led to examine the infinitesimal change produced in a given finite error, as expressed by a given function, by the superposition of a new infinitesimal error; and from the analytical expression arrived at, it is shown how to find the form of the function of error resulting from the combination of an infinite number of given infinitesimal errors. This form is found to be altogether independent of the nature or laws of the component errors. If we assume the following data as known, viz.

$m$=sum of the mean values of the component errors,

$h$=sum of the mean values of the squares of component errors,

$i$=sum of squares of the mean values of component errors,

it is proved that the probability of the actual resulting error being found to lie between $x$ and $x+dx$ is

$$\frac{1}{\sqrt{2\pi\,(h-i)}}\,\epsilon^{-\frac{(x-m)^2}{2(h-i)}}\,dx.$$

This result will be found to agree with Poisson's.

*April* 29, 1869.

Lieut.-General SABINE, President, in the Chair.

Pursuant to notice given at the last Meeting, Alphonse DeCandolle, of Geneva; Charles Eugène Delaunay, of Paris; and Louis Pasteur, of Paris, were ballotted for and elected Foreign Members of the Society.

The following communications were read:—